

ON REALISING $N=1$ SUPER YANG-MILLS IN M THEORY.

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ABSTRACT

Pure $\mathcal{N}=1$ super Yang-Mills theory can be realised as a certain low energy limit of M theory near certain singularities in G_2 -holonomy spaces. For $SU(n)$ and $SO(2n)$ gauge groups these M theory backgrounds can be regarded as strong coupling limits of wrapped D6-brane configurations in Type IIA theory on certain non-compact Calabi-Yau spaces such as the deformed conifold. Various aspects of such realisations are studied including the generation of the superpotential, domain walls, QCD strings and the relation to recent work of Vafa. In the spirit of this recent work we propose a ‘gravity dual’ of M theory near these singularities.

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1. Introduction.

Pure super Yang-Mills theory in four dimensions is an interesting gauge theory where we believe we can say something about the physics at strong coupling. The model shares some of the features of ordinary QCD, but is easier to understand because of the added feature of supersymmetry.

This super Yang-Mills theory can be realised in various limits of string theory and M theory vacua. These are typically limits of the world-volume theories on intersecting or wrapped branes or limits of theories localised near singularities [1, 2, 3, 4, 5, 6, 7, 8]². As we will see, some of these studies are closely related.

In [4] we studied some aspects of M theory physics near certain singularities in G_2 holonomy spaces. Our motivation for this was to begin to answer the question: what is the physics of M theory compactified on a G_2 -holonomy space? The motivation for the question was that its answer might eventually shed some new light on the physics of four dimensional theories with $\mathcal{N}=1$ supersymmetry.

The singularities in question took the form of families of ADE singularities in \mathbb{R}^4 parametrised by a supersymmetric 3-cycle M . In other words, the total space of the singular 7-manifold J took the form of a singular fiber bundle with fibers \mathbb{R}^4/Γ and base M . The zero-section of this bundle, which is a copy of M , is an associative 3-cycle in the singular G_2 -holonomy space ie a supersymmetric 3-cycle. When the volume of M is large the 7-manifold looks approximately like flat $\mathbb{R}^4/\Gamma \times \mathbb{R}^3$ and M theory on $J \times \mathbb{R}^{3,1}$ looks approximately like M theory on $X \equiv \mathbb{R}^4/\Gamma \times \mathbb{R}^{6,1}$. When Γ is a finite ADE subgroup of $SU(2)$ acting on $\mathbb{C}^2 \equiv \mathbb{R}^4$, the low energy physics of M theory on X is described by super Yang-Mills theory on $0 \times \mathbb{R}^{6,1}$, the singular subset of X . We argued that the theory on J should be regarded as a twisted compactification of seven dimensional super Yang-Mills formulated on $M \times \mathbb{R}^{3,1}$. The massless spectrum of the effective four dimensional theory was determined to be $\mathcal{N} = 1$ super Yang-Mills with $b_1(M)$ adjoint chiral multiplets. In particular when $b_1(M)$ is zero the low energy physics of M theory near singularities of this type corresponds to pure super Yang-Mills. To be more precise, the super Yang-Mills description is valid when the volume of M is large compared to the Planck scale and one considers energy scales below the mass of the lowest Kaluza-Klein excitation. We argued that a non-trivial superpotential in this model is generated by fractional $M2$ -brane instantons which wrap M . This potential agrees with the field theoretic result.

For gauge group $SU(n)$ ie $\Gamma \equiv \mathbb{Z}_n$ a natural proposal is that this M theory

² For a review of the approach via branes with many more references see [9]

background is the strong string coupling limit of Type IIA string theory on $T^*(M)$ with n D6-branes wrapped around the zero section [4].

The original motivation for this work was to elaborate on the discussion in [4] by giving a stronger argument for the generation of the superpotential and also to address issues such as domain walls and QCD strings. We will partly achieve this by presenting much simpler explicit examples of the singular M theory background spacetime in which M is a three sphere S^3 . In this case J is a \mathbb{Z}_n -orbifold of the standard spin bundle $S(S^3)$ of S^3 . $S(S^3)$ is topologically trivial ie $S(S^3) \equiv \mathbb{R}^4 \times S^3$, but admits a non-trivial bundle metric with G_2 -holonomy [10, 11]. The quotient by \mathbb{Z}_n which defines J is topologically equivalent to $\mathbb{R}^4/\mathbb{Z}_n \times S^3$ where the \mathbb{Z}_n acts on each fiber as an ADE subgroup of $SU(2)$ acting on $\mathbb{C}^2 \equiv \mathbb{R}^4$. The singularities of J are precisely as described above: a family of A_{n-1} singularities fibered over S^3 . Most importantly, \mathbb{Z}_n preserves the G_2 -structure on $S(S^3)$ so that J has G_2 -holonomy. This construction can be generalised to any ADE subgroup of $SU(2)$. In the $SU(n)$ case however which will be our primary focus in this paper, M theory on J is naturally proposed as the strong coupling limit of Type IIA string theory on $T^*(S^3)$ with n D6-branes wrapping the S^3 . This latter string theory background was recently studied in [8]. As we will see, combining the results of [8] with those presented here gives a strong indication that the proposal is correct.

One of the central themes of [8] was to propose a closed string, gravity dual of the open string theory describing the D6-branes. Given that we have proposed a strong coupling limit of the latter and have gathered evidence in its favour, it is natural to ponder the existence of an M theory description of the gravity dual proposed in [8]. We will make a proposal of just such a dual here. This M theory background has some distinguishing features. For instance there are \mathbb{Z}_n -charge strings which we suggest should be identified with the super QCD-strings in the correspondence with Yang-Mills theory. The model also contains domain walls.

This paper is organised as follows. In the next section we aim to clarify aspects of the superpotential in these models by compactifying the M theory background on a circle and considering Type IIA theory on J . Here we will make contact with some field theory results [12] which will be reinterpreted here in terms of fractional branes. In particular, we will see that the description of fractional branes as wrapped branes [13] is already anticipated in quantum Yang-Mills theory. Next we make some comments concerning BPS-saturated domain walls in these models and tentatively identify domain walls of the expected tension. Finally, in the light of [8], we propose a ‘gravity dual’ of M theory on J .

Whilst this work was in progress we learned of the forthcoming work of

Atiyah, Maldacena and Vafa [14] in which the same models are studied. We wish to thank J. Maldacena for informing us of this work. We would also like to mention the recent paper [15] which discusses smooth supergravity solutions describing branes on special holonomy manifolds (including those described in [10, 11]) and is relevant for a discussion of gravity duals of the corresponding world-volume theories.

2. Type IIA theory on J .

Consider Type IIA string theory compactified to three dimensions on a seven manifold X with holonomy G_2 . If X is smooth we can determine the massless spectrum of the effective supergravity theory in three dimensions as follows. Compactification on X preserves four of the 32 supersymmetries in ten dimensions, so the supergravity theory has three dimensional $\mathcal{N} = 2$ local supersymmetry. The relevant bosonic fields of the ten dimensional supergravity theory are the metric, B -field, dilaton plus the Ramond-Ramond one- and three-forms. These we will denote by g, B, ϕ, A_1, A_3 respectively. Upon Kaluza-Klein reduction the metric gives rise to a three-metric and $b_3(X)$ massless scalars. The latter parametrise the moduli space of G_2 -holonomy metrics on X . B gives rise to $b_2(X)$ periodic scalars φ_i . ϕ gives a three dimensional dilaton. A_1 reduces to a massless vector, while A_3 gives $b_2(X)$ vectors and $b_3(X)$ massless scalars. In three dimensions a vector is dual to a periodic scalar, so at a point in moduli space where the vectors are free we can dualise them. The dual of the vector field originating from A_1 is the period of the RR 7-form on X , whereas the duals of the vector fields coming from A_3 are given by the periods of the RR 5-form A_5 over a basis of 5-cycles which span the fifth homology group of X . Denote these by scalars by σ_i . All in all, in the dualised theory we have in addition to the supergravity multiplet, $b_2(X) + b_3(X)$ scalar multiplets. Notice that $b_2(X)$ of the scalar multiplets contain two real scalar fields, both of which are periodic.

Now we come to studying the Type IIA theory on J . Recall that J is defined as an orbifold of the standard spin bundle of S^3 . To determine the massless spectrum of IIA string theory on J we can use standard orbifold techniques. However, the answer can be phrased in a simple way. J is topologically $\mathbb{R}^4/\mathbb{Z}_n \times S^3$. This manifold can be desingularised to give a smooth seven manifold $M^{\mathbb{Z}_n}$ which is topologically $X^{\mathbb{Z}_n} \times S^3$, where $X^{\mathbb{Z}_n}$ is homeomorphic to an *ALE* space. The string theoretic cohomology groups of J are isomorphic to the usual cohomology groups of $M^{\mathbb{Z}_n}$. The reason for this is simple: J is a global orbifold of $S(S^3)$. The string theoretic cohomology groups count massless string states in the orbifold CFT. The massless string states in the twisted sectors are localised near the fixed points of the action of \mathbb{Z}_n on the spin bundle. Near the fixed points we can work on the tangent space of $S(S^3)$ near a fixed point and the action of \mathbb{Z}_n there is just its natural action on $\mathbb{R}^4 \times \mathbb{R}^3$.

Note that blowing up J to give $M^{\mathbb{Z}_n}$ cannot give a metric with G_2 -holonomy which is continuously connected to the singular G_2 -holonomy metric on J , since this would require that the addition to homology in passing from J to $M^{\mathbb{Z}_n}$ receives contributions from four-cycles. This is necessary since

these are dual to elements of $H^3(M)$ which generate metric deformations preserving the G_2 -structure. This argument does not rule out the possibility that $M^{\mathbb{Z}_n}$ admits ‘disconnected’ G_2 -holonomy metrics, but is consistent with the fact that pure super Yang-Mills theory in four dimensions does not have a Coulomb branch.

The important points to note are that the twisted sectors contain massless states consisting of r scalars and r vectors where r is the rank of the corresponding ADE group associated to \mathbb{F} . In the case of primary interest $r = n - 1$. The r scalars can intuitively thought of as the periods of the B field through r two cycles. In fact, for a generic point in the moduli space of the orbifold conformal field theory the spectrum contains massive particles charged under the r twisted vectors. These can be interpreted as wrapped D2-branes whose quantum numbers are precisely those of W -bosons associated with the breaking of an ADE gauge group to $U(1)^r$. This confirms our interpretation of the origin of this model from M theory: the values of the r B -field scalars can be interpreted as the expectation values of Wilson lines around the eleventh dimension associated with this symmetry breaking. At weak string coupling and large S^3 volume these states are very massive and the extreme low energy effective dynamics of the twisted sector states is described by $\mathcal{N}=2$ $U(1)^r$ super Yang-Mills in three dimensions. Clearly however, the underlying conformal field theory is not valid when the W -bosons become massless. The appropriate description is then the pure super Yang-Mills theory on $\mathbb{R}^3 \times S^1$ which corresponds to a sector of M theory on $J \times S^1$. In this section however, our strategy will be to work at a generic point in the CFT moduli space which corresponds to being far out along the Coulomb branch of the gauge theory. We will attempt to calculate the superpotential there and then continue the result to four dimensions. This exactly mimics the strategy of [16, 12] in field theory and [3] in the context of F -theory. Note that we are implicitly ignoring gravity here. More precisely, we are assuming that in the absence of gravitational interactions with the twisted sector, the low energy physics of the twisted sectors of the CFT is described by the Coulomb branch of the gauge theory. This is natural since the twisted sector states are localised at the singularities of $J \times \mathbb{R}^{2,1}$ whereas the gravity propagates in bulk.

In this approximation, we can dualise the photons to obtain a theory of r chiral multiplets, each of whose bosonic components (φ and σ) is periodic. But remembering that this theory arose from a non-Abelian one we learn that the moduli space of classical vacua is

$$\mathcal{M}_{cl} = \frac{\mathbb{C}^r}{\Lambda_W^{\mathbb{C}} \rtimes W_g} \quad (1)$$

where $\Lambda_W^{\mathbb{C}}$ is the complexified weight lattice of the ADE group and W_g is the Weyl group.

We can now ask about quantum effects. In particular is there a non-trivial superpotential for these chiral multiplets? In a theory with four supercharges BPS instantons with only two chiral fermion zero modes can generate a superpotential. Are there instantons in Type IIA theory on J ? BPS instantons come from branes wrapping supersymmetric cycles and Type IIA theory on a G_2 -holonomy space can have instantons corresponding to D6-branes wrapping the space itself or D2-brane instantons which wrap supersymmetric 3-cycles. For smooth G_2 -holonomy manifolds these were studied in [17]. In the case at hand the D6-branes would generate a superpotential for the dual of the graviphoton multiplet which lives in the gravity multiplet but since we wish to ignore gravitational physics for the moment, we will ignore these. In any case, since J is non-compact, these configurations have infinite action. The D2-branes on the other hand are much more interesting. They can wrap the supersymmetric S^3 over which the singularities of J are fibered. We can describe the dynamics of a wrapped D2-brane as follows. At large volume, where the sphere becomes flatter and flatter the world-volume action is just the so called ‘quiver gauge theory’ described in [18]. Here we should describe this theory not just on S^3 but on a supersymmetric S^3 embedded in a space with a non-trivial G_2 -holonomy structure. The upshot is that the world-volume theory is in fact a cohomological field theory [19, 20] so we can trust it for any volume as long as the ambient space has G_2 -holonomy. Note that, since we are ignoring gravity, we are implicitly ignoring higher derivative corrections which could potentially also affect this claim. Another crucial point is that the S^3 which sits at the origin in \mathbb{R}^4 in the covering space of J is the supersymmetric cycle, and the spheres away from the origin are not supersymmetric, so that the BPS wrapped D2-brane is constrained to live on the singularities of J . In the quiver gauge theory, the origin is precisely the locus in moduli space at which the single D2-brane can fractionate (according to the quiver diagram) and this occurs by giving expectation values to the scalar fields which parametrise the Coulomb branch which corresponds to the position of our D2-brane in the dimensions normal to J .

What contribution to the superpotential do the fractional D2-branes make? To answer this we need to identify the configurations which possess only two fermionic zero modes. We will not give a precise string theory argument for this, but using the correspondence between this string theory and field theory will identify exactly which D-brane instantons we think are responsible for generating the superpotential. This may sound like a strong assumption, but as we hope will become clear, the fact that the fractional D2-branes are wrapped D4-branes is actually anticipated by the field the-

ory! This makes this assumption, in our opinion, somewhat weaker and adds credence to the overall picture being presented here.

In the seminal work of [13], it was shown that the fractionally charged D2-branes are actually D4-branes which wrap the ‘vanishing’ 2-cycles at the origin in \mathbb{R}^4/Γ . More precisely, each individual fractional D2-brane, which originates from a single D2-brane possesses D4-brane charge, but the total configuration, since it began life as a single D2-brane has zero D4-brane charge. The possible contributions to the superpotential are constrained by supersymmetry and must be given by a holomorphic function of the r chiral superfields and also of the holomorphic gauge coupling constant τ which corresponds to the complexified volume of the S^3 in eleven dimensional M theory. We have identified above the bosonic components of the chiral superfields above. τ is given by

$$\tau = \int \varphi + iC \quad (2)$$

where φ is the G_2 -structure defining 3-form on J . The period of the M theory 3-form through S^3 plays the role of the theta angle.

The world-volume action of a D4-brane contains the couplings

$$L = B \wedge A_3 + A_5 \quad (3)$$

Holomorphy dictates that there is also a term

$$B \wedge \varphi \quad (4)$$

so that the combined terms are written as

$$B \wedge \tau + A_5 \quad (5)$$

Since the fourbranes wrap the ‘vanishing cycles’ and the S^3 we see that the contribution of the D4-brane corresponding to the k -th fractional D2 takes the form

$$S = -\beta_k \cdot z \quad (6)$$

where we have defined

$$z = \tau\varphi + \sigma \quad (7)$$

and the β_k are charge vectors. The r complex fields z are the natural holomorphic functions upon which the superpotential will depend.

The wrapped D4-branes are the magnetic duals of the massive D2-branes which we identified above as massive W -bosons. As such they are magnetic monopoles for the original $SU(n)$ gauge symmetry. Their charges are therefore given by an element of the co-root lattice of the Lie algebra and thus

each of the $r + 1$ β 's is a rank r vector in this space. Choosing a basis for this space corresponds to choosing a basis for the massless states in the twisted sector Hilbert space which intuitively we can think of as a basis for the cohomology groups Poincare dual to the 'vanishing' 2-cycles. A natural basis is provided by the simple co-roots of the Lie algebra of $SU(n)$, which we denote by α_k^* for $k = 1, \dots, r$. This choice is natural, since these, from the field theory point of view are the fundamental monopole charges.

At this point it is useful to mention that the r wrapped D4-branes whose magnetic charges are given by the simple co-roots of the Lie algebra correspond in field theory to monopoles with charges α_k^* and each of these is known to possess precisely the right number of zero modes to contribute to the superpotential. Since we have argued that in a limit of the Type IIA theory on J , the dynamics at low energies is governed by the field theory studied in [12] it is natural to expect that these wrapped fourbranes also contribute to the superpotential. Another striking feature of the field theory is that these monopoles also possess a fractional instanton number - the second Chern number of the gauge field on $\mathbb{R}^3 \times S^1$. These are precisely in correspondence with the fractional D2-brane charges. Thus, in this sense, the field theory anticipates that fractional branes are wrapped branes.

In the field theory on $\mathbb{R}^3 \times S^1$ it is also important to realise that there is precisely one additional BPS state which contributes to the superpotential. The key point is that this state, unlike the previously discussed monopoles have dependence on the periodic direction in spacetime. This state is associated with the affine node of the Dynkin diagram. Its monopole charge is given by

$$-\sum_{k=1}^r \alpha_k^* \quad (8)$$

and it also carries one unit of instanton number.

The action for this state is

$$S = \sum_{k=1}^r \alpha_k^* \cdot z - 2\pi i \tau \quad (9)$$

Together, these $r + 1$ BPS states can be regarded as fundamental in the sense that all the other finite action BPS configurations can be thought of as bound states of them.

Thus, in the correspondence with string theory it is also natural in the same sense as alluded to above that a state with these corresponding quantum numbers also contributes to the superpotential. It may be regarded as a bound state of anti-D4-branes with a charge one D2-brane. In the case of $SU(n)$ this is extremely natural, since the total D4/D2-brane charge of the $r + 1$ states is zero/one, and this is precisely the charge of the D2-brane configuration on S^3 whose world-volume action is the quiver gauge theory for the

affine Dynkin diagram for $SU(n)$. In other words, the entire superpotential is generated by a single D2-brane which has fractionated.

In summary, we have seen that the correspondence between the Type IIA string theory on J and the super Yang-Mills theory on $\mathbb{R}^3 \times S^1$ is quite striking. Within the context of this correspondence we considered a smooth point in the moduli space of the perturbative Type IIA CFT, where the spectrum matches that of the Yang-Mills theory along its Coulomb branch. On the string theory side we concluded that the possible instanton contributions to the superpotential are from wrapped D2-branes. Their world volume theory is essentially topological, from which we concluded that they can fractionate. As is well known, the fractional D2-branes are really wrapped fourbranes. In the correspondence with field theory, the wrapped fourbranes are magnetic monopoles, whereas the D2-branes are instantons. Thus if, these branes generate a superpotential they correspond, in field theory to monopole-instantons. This is exactly how the field theory potential is known to be generated. We thus expect that the same occurs in the string theory on J .

Finally, the superpotential generated by these states is of affine-Toda type and is known to possess n minima corresponding to the vacua of the $SU(n)$ super Yang-Mills theory on \mathbb{R}^4 . The value of the superpotential in each of these vacua is of the form $e^{\frac{2\pi i \tau}{n}}$. As such it formally looks as though it was generated by fractional instantons, and in this context fractional $M2$ -brane instantons. This result holds in the four dimensional M theory limit because of holomorphy and thus elaborates upon the result of [4].

3. On Domain Walls in M theory on J .

We now turn our attention to domain walls. Since the model has a finite number of vacua one might expect that there are domain walls which separate them. This is believed to be the case for the super Yang-Mills theory because that theory has a spontaneously broken discrete symmetry. This is the quantum remnant of the classical $U(1)$ R -symmetry. M theory on J (or any other G_2 -holonomy manifold) does not obviously have a $U(1)$ symmetry. The classical supergravity theory does have a \mathbb{Z}_2 R -symmetry which acts as ± 1 on the supercharges and preserves the G_2 -structure. This order two symmetry is naturally identified with the \mathbb{Z}_2 symmetry of the super Yang-Mills vacua. Following this line of reasoning does not give us any obvious reason to expect to see domain walls connecting the various vacua. However, the model does appear to possess domain walls with some of the right properties, as we will see.

As discussed in [22], in M theory compactification on a G_2 -holonomy

manifold the BPS domain walls are basically of two types. Firstly there are $M5$ -branes which wrap supersymmetric three cycles. Secondly there are $M2$ -branes which sit at a point on the seven manifold. Actually, the general story is more complicated since $M2$ -branes and $M5$ -branes can form bound states and one must not rule out the possibility of bound state walls in the compactified theory. This is in line with the fact that the central charge in the $\mathcal{N} = 1$ supersymmetry algebra in four dimensions which represents the charge of domain walls is complex.

In M theory on J the S^3 is a supersymmetric cycle, so an $M5$ -brane wrapped on it looks like a domain wall. Because it is BPS saturated its tension is given by the volume

$$T = Vol(S^3) = \frac{1}{g_{YM}^2} \quad (10)$$

which is identified as the gauge coupling in the correspondence with Yang-Mills. One important aspect of the super Yang-Mills theory is that the t'Hooft large n limit is a good approximation to its physics. This means that

$$T \approx n \quad (11)$$

and this is the expected behaviour of the domain walls in this model [2]. The $M2$ -brane domain walls do not have a tension which depends on the coupling. However, the $M2$ - $M5$ -bound state wall has a tension whose square is given by the sum of the squares of its two constituents. In the large n limit the tension of this bound state is also of order n , so we cannot distinguish the pure $M5$ -brane from the bound state by this reasoning. So, even though M theory on J does not appear to undergo discrete symmetry breaking, BPS domain walls appear to exist with at least some of the expected properties of their super Yang-Mills counterparts.

In the IIA theory on J the wrapped $M5$ -brane domain walls go over to $D4$ -branes wrapping the supersymmetric S^3 . This is natural, since the various vacua of the three dimensional theory are related by shifts of the σ . These latter fields are just the twisted RR scalars which can be thought of as periods of the RR 5-form potential through the vanishing 5-cycles. Since these fields couple naturally to the $D4$ -branes we see that the fourbrane charges shift by one unit from vacuum to vacuum. This lends further support to the claim that the domain walls in M theory on J are $M5$ -branes - possibly bound to $M2$ -branes.

4. ‘Gravity’ Dual of M theory on J .

As we mentioned in the introduction, M theory on J is naturally proposed as the strong string coupling limit of Type IIA theory on $T^*(S^3)$ with n D6-branes wrapped around the S^3 .

The IIA background consisting of n D6-branes wrapped around the S^3 in the deformation of the conifold singularity $T^*(S^3)$ was studied by Vafa in [8]. In this work it was shown how open topological string amplitudes can be used to calculate certain higher derivative terms in the effective world-volume action on \mathbb{R}^4 . Moreover a gravity dual of this model was proposed. This was defined as Type IIA theory on the resolution of the conifold singularity together with RR fluxes. This smooth non-compact Calabi-Yau background can loosely be regarded as an \mathbb{R}^4 -bundle over S^2 . As regards the fluxes it is crucial that there are n units of RR 2-form flux through the S^2 corresponding to the D6-brane charge. An important part of [8] was that the closed string topological amplitudes are not modified in the presence of RR flux. The two sets of topological amplitudes agreed if the complexified Kahler class of the $\mathbb{P}^1 = S^2$ on the gravity side was identified with the gaugino bilinear superfield on the open string side. In the low energy limit of the D6-brane theory it was shown how the effective superpotential for the super Yang-Mills theory emerged from these topological amplitudes and on the closed string side these corresponded to world-sheet instantons wrapping the S^2 . In fact, on the open string side these have the form of fractional D2-brane instantons wrapping the S^3 . This is in agreement with what we found in the M theory limit, since these become fractional $M2$ -branes. Thus the result of [8] is consistent with the results of this paper and strongly support the claim that M theory on J is the strong coupling limit of the wrapped D6-brane system. In fact, the amplitudes discussed in [8] are well defined for all values of the string coupling constant, including the strong coupling limit. Thus they are also describing the physics of M theory on J .

The previous statement is true regardless of the existence of a gravity dual of the wrapped D6-brane system on $T^*(S^3)$. However, it would be interesting to extend the ‘duality’ between the wrapped D6-brane system and M theory on J to the gravity dual of the wrapped D6-brane system and another M theory background. One way of phrasing this desire is to ask if there is an M theory description of the IIA theory on the resolved conifold with RR fluxes? We can proceed to answer this question as follows.

Consider the resolved conifold with only the n units of RR 2-form flux turned on over the S^2 . Because this flux is non-trivial it implies that the M theory circle is fibred over the S^2 giving a circle bundle of first Chern number n . Thus a natural proposal for the M theory background is an \mathbb{R}^4 - bundle

over S^3/\mathbb{Z}_n - because the latter space indeed is the total space of a degree n bundle over S^2 . Moreover, at infinity this space looks just like infinity in J . We can now propose a dual of the IIA theory on the resolved conifold with only RR 2-form flux: M theory on $\tilde{J} \equiv S(S^3/\mathbb{Z}_n)$ is the standard spin bundle of the Lens space. This space has a natural G_2 -holonomy metric [10] and in fact M theory on \tilde{J} has the same symmetries as M theory on J .

A crucial point of [8] appeared to be the requirement of RR 4-form and 6-form fluxes in the background. These correspond to background values of the four-form field strength G in M theory on \tilde{J} . The author is not aware of how to turn on G in a G_2 -holonomy background without breaking supersymmetry. The arguments in favour of this have always been in the context of compactifying to Minkowski space [21, 22]³. However, it is plausible that this can be done if the four dimensional spacetime normal to the seven manifold is anti de Sitter. This would be in accordance with the fact that the value of the superpotential in the super Yang-Mills vacuum is non-zero and therefore when embedded into a theory containing gravity contributes to a negative cosmological constant in the vacuum. Our lack of understanding of this point also reflects the fact that we essentially ignored gravity when studying M theory on J .

However, we have certainly identified what we consider to be a good candidate for the M theory background dual to IIA theory on the resolved conifold with background RR 2-form fluxes.

The proposed model (and - if it exists - the model with G -flux) seem to have some quite striking features which would be required of a dual description of M theory on J . For instance the theory contains \mathbb{Z}_n -charged strings which arise from wrapping the $M2$ -brane around non-trivial 1-cycles which generate the fundamental group of the Lens space. These are naturally identified with the super QCD strings.

The corresponding Type IIA model in three dimensions has BPS domain walls corresponding to D4-branes wrapping the Lens space. Since the fundamental group of the Lens space is \mathbb{Z}_n there are n classical vacua of the D4-brane world-volume theory corresponding to the n inequivalent irreducible representations of \mathbb{Z}_n . These are n distinct domain walls and their spectrum - because they are BPS states - is apparently the same in the quantum string theory [24].

In the M theory limit these correspond to $M5$ -branes wrapping the Lens space. These apparently have non-trivial flat B -field connections on their world volumes. If we now consider ‘reducing’ M theory on \tilde{J} to give the IIA

³In [21] this is a supergravity calculation. In [22] this is based upon a natural proposal for the superpotential of the compactified theory [23].

theory on the resolved conifold with n units of RR 2-form flux (as opposed to considering IIA theory on \tilde{J}) then these $M5$ -brane domain walls go over to D4-brane domain walls which wrap the S^2 . These D4-branes were identified in [8] with the domain walls corresponding to those expected in the super Yang-Mills theory.

One puzzling point however is that one only expects to see $n - 1$ distinct domain walls in the super Yang-Mills theory, whereas here we appear to find n . The remainder of this paragraph is devoted to some speculative remarks about this issue. In the Type IIA theory on \tilde{J} , $n - 1$ of these walls is certainly distinguished, since they carry non-zero \mathbb{Z}_n quantum numbers. The trivially charged domain wall on the other hand can be thought of as a wrapped D4-brane on $S(S^3)$ which is \mathbb{Z}_n invariant. In the Type IIA theory on $S(S^3)$ this domain wall can be regarded as separating flat three dimensional Minkowski space from three dimensional anti de Sitter space or perhaps two anti de Sitter regions. M theory on $S(S^3)$ is the gravity dual of M theory on $S(S^3)$! Thus, the additional wall in the theory on \tilde{J} corresponding to the affine node appears to map in the theory on J to a domain wall separating two regions of empty space in M theory on J , and it is unnatural to identify these with vacua of super Yang-Mills. Perhaps there is a more rigorous argument which identifies the $n - 1$ domain walls.

Most of the results of this paper can be generalised to theories with $SO(2n)$ and E_n symmetries by replacing \mathbb{Z}_n with the D and E type subgroups of $SU(2)$.

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